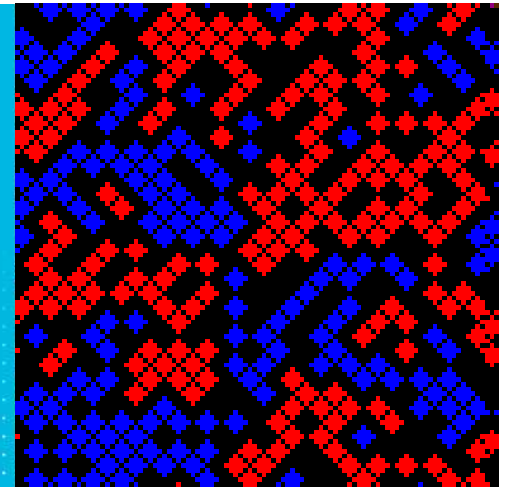




Clean-Slate Design of MANETs via Locally Coupled Particle Systems (LCPS)

Subtopic Area: Theory



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Outline

- LCPS model
- LCPS (asymptotic) solution -- algorithm
- Application Case 1: Single-Hop Random Access Networks
- Application Case 2: OFDMA Cellular Wireless Networks
- Application Case 3: Large-Scale MANETs
 - Resource sharing: Channel selection, client association, scheduling, power allocation
 - Routing: Backbone routing, geographic routing and other schemes
- Proposed plan
 - Asymptotic analysis for *given* routing
 - Simulation to validate and begin integration with networking protocol

Distributed Optimization in Locally-Coupled Systems: Methodology

- Consider networked system represented by graph G with nodes $v \in V$ and edges $(v, w) \in E$
- Goal: Maximize global objective function which is the sum of local utilities:

$$\max_{x \in S} U(x), \text{ where } U(x) = \sum_{v \in V} u_v(x_{N_v^+})$$

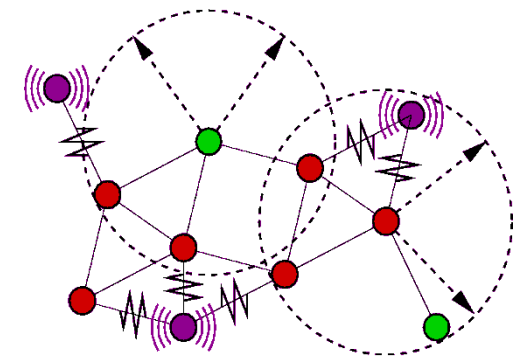
$u_v(x_{N_v^+})$ = utility of node v depends only on its state x and its neighbors, i.e., $N_v^+ = \{w : (v, w) \in E\} \cup \{v\}$

- $P(x) \sim \exp\{\beta U(x)\}$ results in a Markov Random Field, whose equilibrium distribution furnishes powerful insight for identifying the optimal solution:

$U(x)$ is maximized as a limit of Gibbs measure

$$P_\beta(x) = \exp\{\beta U(x)\} / \sum_{s \in S} \exp\{\beta U(s)\} \Rightarrow$$

$$\lim_{\beta \rightarrow \infty} P_\beta(x^*) = 1 \text{ for } x^* = \arg \max_{x \in S} U(x)$$



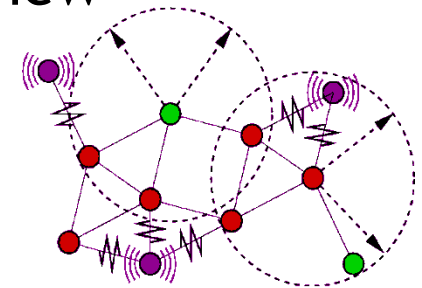
Distributed Optimization in Locally-Coupled Systems: Optimal Solution

- Global optimum for $U(x)$ is obtained with high likelihood by sampling from Gibbs distribution, $P_\beta(x)$, as follows :

-- Pick a node v in V

-- Given current state x_{-v} of all other nodes, select new state x_v with probability

$$P_\beta(x_v | x_{-v}) \leftarrow P_\beta(x_v, x_{-v}) / \sum_{y_v} P_\beta(y_v, x_{-v})$$



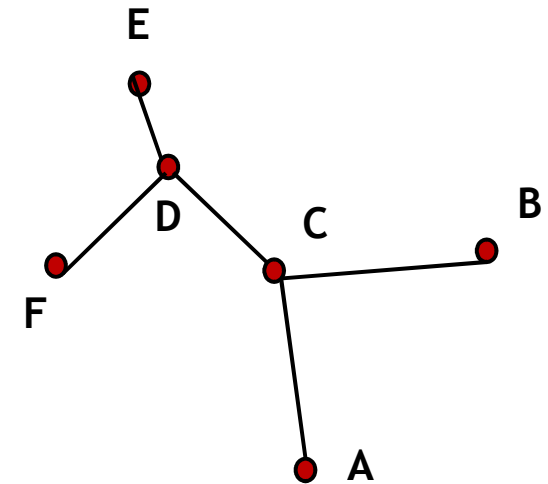
- We recently showed* that in a LCPS computing $P_\beta(x_v | x_{-v})$ requires only knowledge of the two-tier neighborhood structure
- Generates sequence of random variables converging to above Gibbs distribution, and hence yields global optimum for $U(x)$ for large values of β a.s.

* S. Borst, M. Markakis, and I. Saniee, "Non-concave utility maximization in locally coupled systems, with applications to wireless and wireline networks," to appear IEEE/ACM Transactions on Networking, 2013.

Simple Example: Single Channel with 2 Power Levels -- 1

Neighborhoods

- $N_A^+ = \{C\} \cup \{A\}$, $N_B^+ = \{C\} \cup \{B\}$, $N_C^+ = \{A, B, D\} \cup \{C\}$
- $N_E^+ = \{D\} \cup \{E\}$, $N_F^+ = \{D\} \cup \{F\}$, $N_D^+ = \{E, F, C\} \cup \{D\}$



States

- Local state, power levels: $P_A \in \{0, p\}$, *etc.*
- Neighborhood state, power levels: $P_{N_A^+} = P_{\{A,C\}} = (P_A, P_C)$,
 $P_{N_C^+} = P_{\{A,B,D,C\}} = (P_A, P_B, P_C, P_D)$, *etc.*

Local utility functions depend on neighborhood states

- $u_A(P_A, P_C)$, $u_B(P_B, P_C)$, $u_C(P_C, P_A, P_B, P_D)$, $u_E(P_E, P_D)$, $u_F(P_F, P_D)$, $u_D(P_D, P_E, P_F, P_C)$

where for a log utility function we have

$$u_C(P_{N_C^+}) = u_C(P_C, P_A, P_B, P_D) = \log\left(\log\left(1 + \underbrace{\frac{g_C P_C}{g_A P_A + g_B P_B + g_D P_D + \eta}}_{\text{rate}}\right)\right)$$

Simple Example: Single Channel with 2 Power Levels -- 2

$$\text{Total utility : } U(\mathbf{P}) = \sum_{v \in V} u_v(P_{N_v^+})$$

Solution

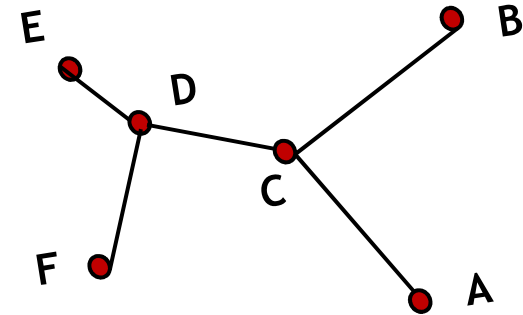
$$P_A(t+1) = \begin{cases} 0 \\ p \end{cases} \text{ with probabilities } \begin{aligned} &\sim e^{\beta(U(A \text{ picks power 0 \& all other nodes pick current power}))} \\ &\sim e^{\beta(U(A \text{ picks power p \& all other nodes pick current power}))} \end{aligned}$$

$$= \begin{cases} 0 \\ p \end{cases} \text{ with probabilities } \begin{aligned} &\sim e^{\beta(U(P_A=0, P_{-A}(t)))} \\ &\sim e^{\beta(U(P_A=p, P_{-A}(t)))} \end{aligned}$$

By local coupling

$$P_A(t+1) = \begin{cases} 0 \\ p \end{cases} \text{ with probabilities } \begin{aligned} &\sim e^{\beta(u_A(0, P_{N_A}(t)) + u_C(0, P_{N_C^+ \setminus \{A\}}(t)))} \\ &\sim e^{\beta(u_A(p, P_{N_A}(t)) + u_C(p, P_{N_C^+ \setminus \{A\}}(t)))} \end{aligned}$$

P_A updates requires states of B , C and D but not E & F , etc.



What Does LCPS Tell Us That We Didn't Know Before?

LCPS massively reduces the state for computation of $U(\mathbf{P}) = \sum_{v \in V} u_v(P_{N_v^+})$

because:

1. Gibbs-type procedure requires computation of $e^{\beta(U(P_A=0, P_{-A}))}$ for (randomized) state update

2. But by LCPS

$$U(P_A, P_{-A}) = u_A(P_A, \sum_{v \in N_A^+} u_v(P_v))$$

$$= u_A(P_A, P_{N_A}) + \underbrace{\sum_{v \sim A} u_v(P_{N_v^+})}_{\text{Two-tier neighborhood of A}} + U_{-A}(\text{independent of A})$$

Two-tier neighborhood of A

thus updates can be computed by looking at only 2-hop neighborhoods

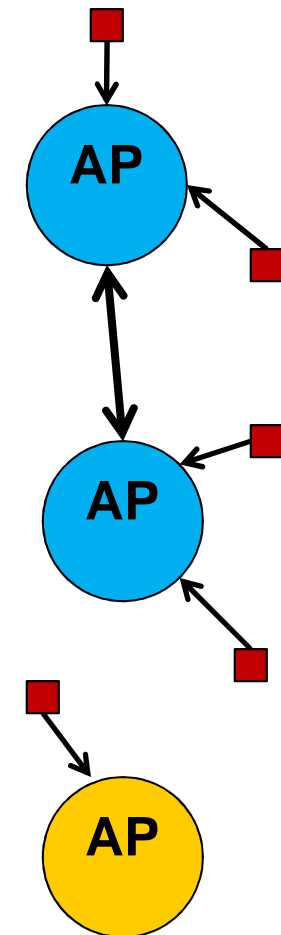
3. Also, to compute probabilities we don't need U_{-A} (independent of A)

Application Case 1: Single-Hop Random Access Multiple AP

- A system consisting of several access points (APs) and clients operating in a number of channels
- Goal: Efficient resource allocation to clients in fair manner with minimal coordination among APs
- Optimization problem:

$$\max U = \sum w_i \log r_i$$

- r_i = throughput of client i
- well-known to achieve proportional fairness
- Resource allocation involves:
 - Choosing the channel to operate in for each AP (**Channel Selection**)
 - Choosing the AP to associate with for each client (**Client Association**)
 - Choosing channel access rate for each AP (**Access**)
 - Scheduling clients for each AP (**Scheduling**)



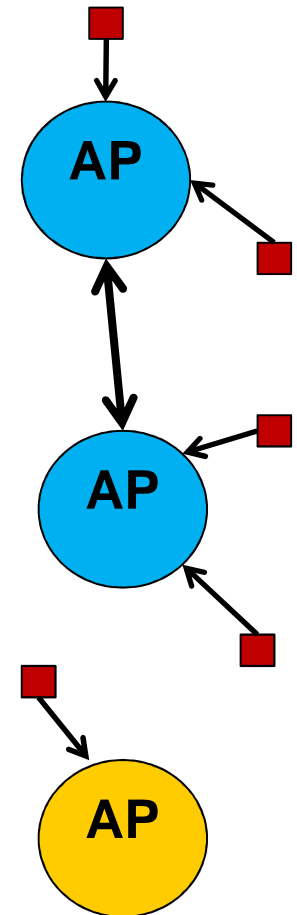
Application Case 1: Solution Overview

- Optimal proportional fairness achieved by jointly solving 4 problems
- Scheduling and Access are updated on fast time scale
- Client Association (CA) & Channel Selection (CS) on slow time scale
- Scheduling & Access can be solved analytically
- For CA and CS, need to consider how the solutions affect Scheduling and Access - non convex
- Solved through Gibbs sampler
 - Maximize system utility: $U = \sum w_i \log r_i$
 - APs/clients make decisions locally and randomly, favoring those resulting in better utility
 - CA: $p(i,n) \sim \frac{B_{i,n,c(n)}}{z_{-i}^n} \prod_{o \in \mathcal{M}^n(\psi)} \frac{z^o}{z^o + w^o}$

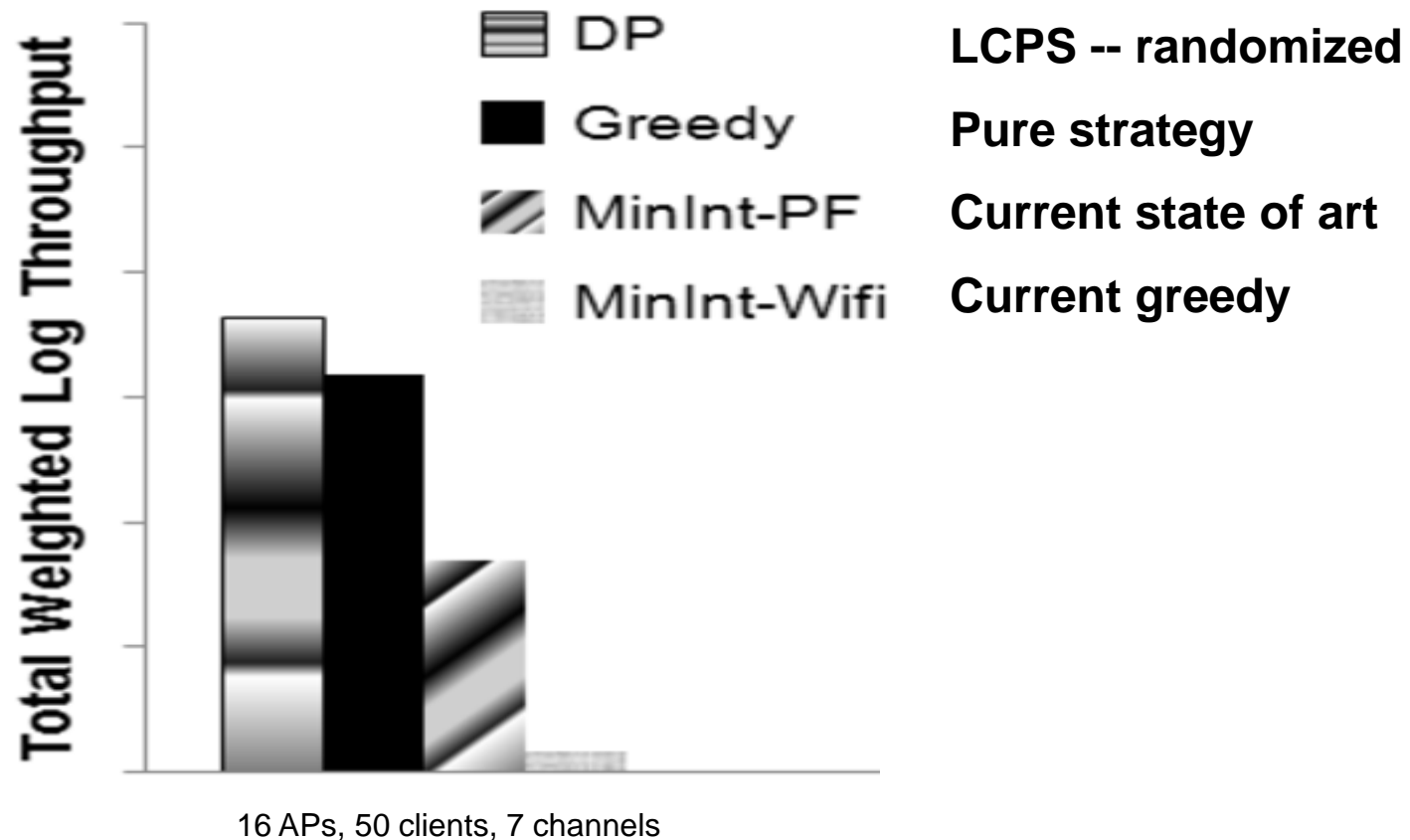
Data Rate

Interference

Channel Congestion
 - Converges to global optimum allocation a.s.
 - Greedy policy for faster convergence
- Traditional approaches based on separation - sub-optimal



Application Case 1: Key Performance Benchmarks



I. Hou and P. Gupta, "Distributed resource allocation for proportional fairness in multi-band wireless systems," Procs. 2011 IEEE Intl. Symposium on Information Theory, St. Petersburg, July 31-Aug 5, 2011.

Application Case 2: OFDMA Cellular Wireless Networks

- System consisting of several possibly interfering access points (APs) and clients operating on multiple frequencies
- Maximize aggregate throughput utility of clients with minimal exchange of state information among APs
- Optimization problem:

$$\max \sum U(r_i)$$

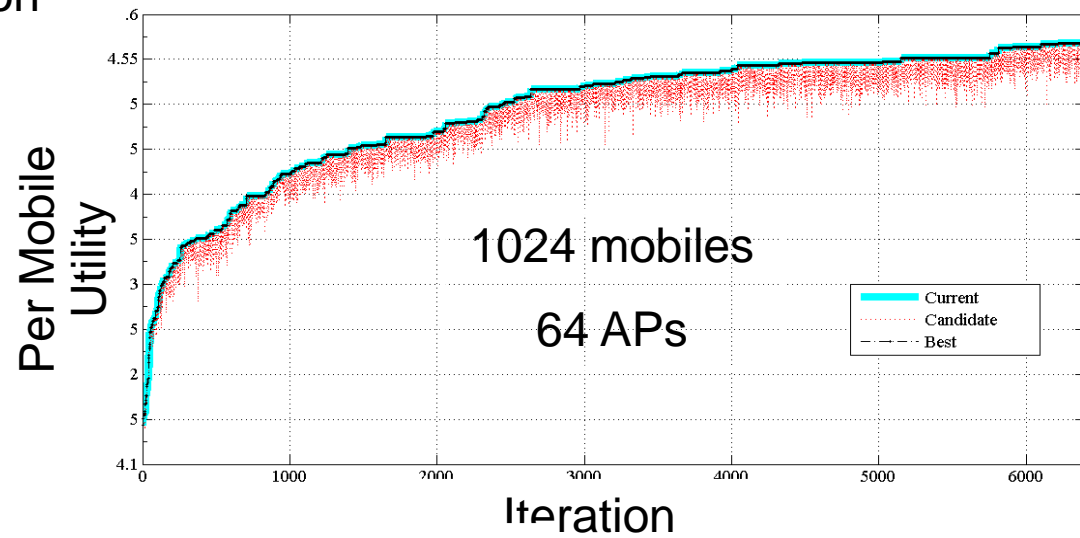
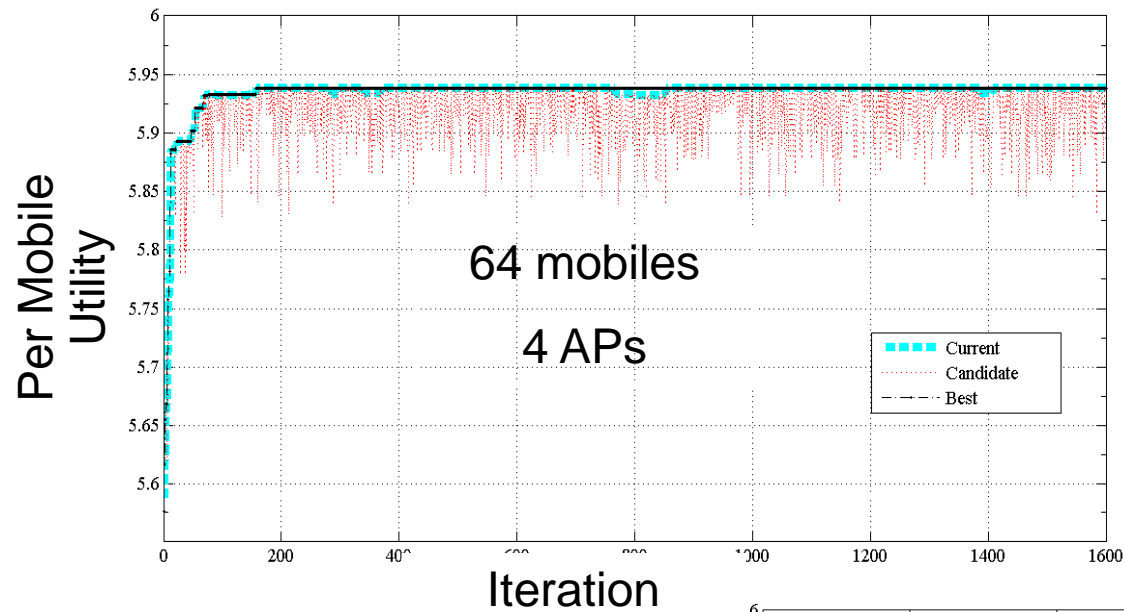
$$s.t. \ r_i \geq r_{i,min}$$

- r_i = throughput of client i
- $r_{i,min}$ = minimum throughput requirement of client i
- $U()$ = arbitrary (possibly non-concave) throughput utility function
- Resource allocation involves three interrelated decisions:
 - Allocating **power levels** to various frequencies at each AP (Power allocation)
 - Selecting **AP to associate** with for each client (Client association)
 - Assigning **time allotments** to various clients at each AP (Scheduling)

Application Case 2: Solution Algorithm

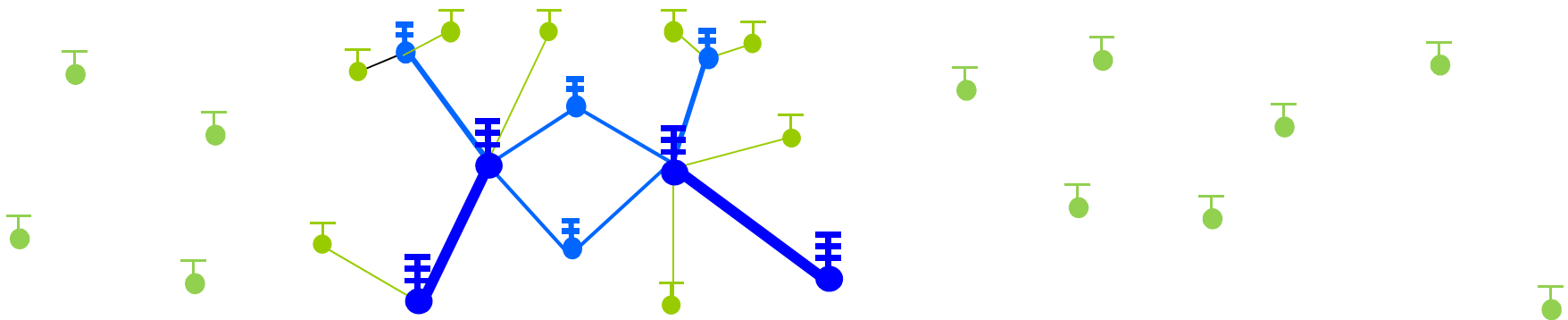
- Three sets of decision variables:
 - P_{jk} : power level allocated by AP k to frequency j
 - Q_{ik} : binary variable whether or not client i associates with AP k
 - T_{ijk} : time allotment assigned to client i on frequency j by AP k
- Optimization is mixed-integer possibly non-concave problem
- Solution algorithm consists of three components:
 - APs update power allocations taking into account impact on neighboring APs due to interference
 - Clients update association decisions taking into account impact on APs due to congestion
 - APs maintain time fractions through scheduling in strictly local fashion
- Experimental results for up to 16 APs, 16 frequencies and 1024 clients show relatively swift convergence to stable operating point

Application Case 2: Experimental Results



Case 3: Application to Massive MANETs Using Dynamic Reconfigurable Backbone

- Establish and maintain a dynamic backbone
 - In heterogeneous settings, utilizing higher-power nodes
 - Each node is within a single hop of the backbone



- Apply LCPS-based approaches discussed earlier for efficient transfer of traffic to/from nodes to the backbone
 - Backbone nodes act as APs to other nodes in their neighborhood
- Efficient design of backbone
 - Number of approaches in literature including geographic routing

Case 4: Application to Massive MANETs with Assigned (Geographic) Routing

- We know from mobile, email, and other communication that the total number of interactions between N nodes is typically not $O(N^2)$ but $O(N)$
- Depending on geometry of MANET, the average number of end-to-end flows per link varies from $O(1)$ to $O(N)$ – square grid to tree topologies
- In case of $O(1)$ each node needs to
 - Keep track of state for a small number of flows
 - Forward packets/content to next hop according to geographic route
- Rate control may be achieved in multiple ways
 - Associating a virtual node with every link and updating end-to-end rates according to a Gibbs-like scheme for general utility functions
 - Using backpressure via queue length in forwarding packets for concave utility functions
- This is now similar to the previous cases where each node makes a randomized decision on power, channel and which packet to schedule according to Gibbs-like distribution

Next Steps

- Linkage between LCPS with routing (backbone, geographic or other protocols) to determine convergence rate, which needs to be quantified
- Report on scalability of resulting LCPS-based MANET to 1000s of nodes via asymptotic and numerical analysis of joint LCPS and routing mechanism and evaluate convergence rate
- Depending on outcome of above, collaborate on a protocol design that incorporates LCPS and routing and simulate on a high fidelity platform

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